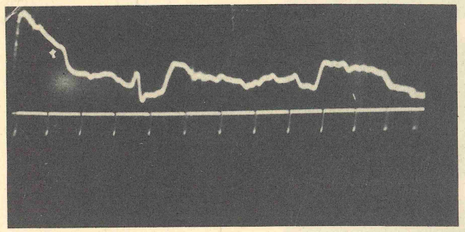
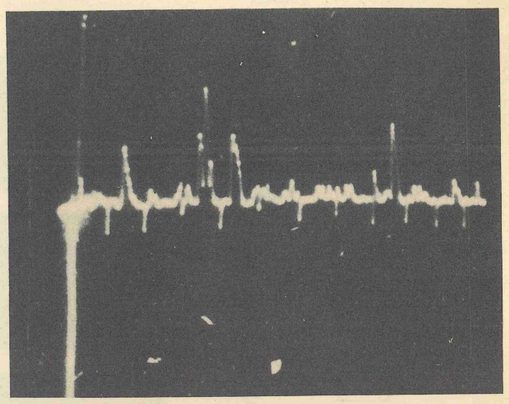


VOLTAGE ON B PHASE

(a)

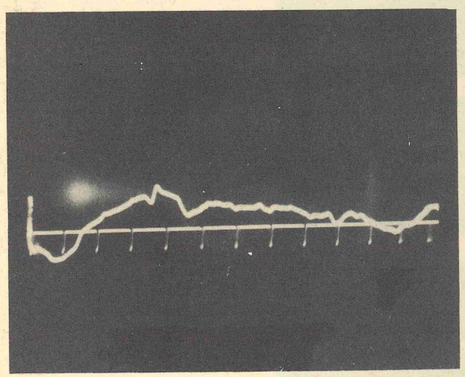


FAULT LOCATOR

(b)

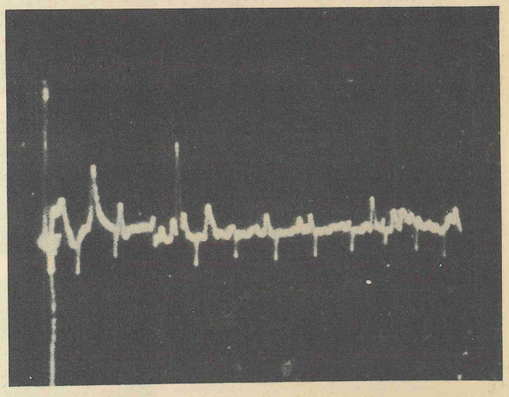
FIGURE 8

FAULT FROM A TO B PHASE
RECORDS TAKEN AT 87 MILES



VOLTAGE ON B PHASE

(a)



FAULT LOCATOR

(b)

FIGURE 9

FAULT FROM B PHASE TO GROUND

AIEE
Technical Paper
51-312

July 1951
Litho in USA

Equip. - Testing

TRAVELING WAVE RELATIONS APPLICABLE TO POWER-SYSTEM FAULT LOCATORS

Laurel J. Lewis
Member AIEE
University of Washington
Seattle, Wash.

Advance Copy
Not Released for Publication

All Rights Reserved by the
American Institute of Electrical Engineers
33 West 39th Street, New York 18, N.Y.

A paper recommended by the AIEE Committee on Transmission and Distribution and approved by the AIEE Technical Program Committee for presentation at the AIEE Pacific General Meeting, Portland, Oregon, August 20-23, 1951. Manuscript submitted May 22, 1951; made available for printing July 10,1951.

PRICE: 30¢ To Members
60¢ To Nonmembers

TRAVELING WAVE RELATIONS APPLICABLE TO POWER-SYSTEM FAULT LOCATORS

By

Laurel J. Lewis

Summary

Fault locators employing traveling waves to measure distance have a response dependent on the propagation characteristics of multi-velocity components of a surge, and the type of fault. Analysis of wave characteristics leads to a method for determining the response with different types of discontinuities.

Introduction

Sectionalizing and switching of lines and the proper repair of facilities requires rapid and accurate location of faults on power systems; thus the development of automatic fault locators is an important activity for improving service continuity.

Three types of fault locators, previously described,[1,2] have been developed by the Bonneville Power Administration and designated as types A, B, and C. A fourth type,[3] the Linascope, differs somewhat from the other three. In each case, performance of the fault locator is conditioned on the transient characteristics of traveling waves; however, the significant problems are not the same for the different systems, because of differences in principles of operation. These principles are illustrated simply in Figure 1; a subsequent analysis will present the characteristics of waves applicable to each system.

Fault Locator Operation

The Type A fault locator operates directly on the line surges generated by a fault. An initial surge travels to the end of the line where the locator is situated and is used to trigger the sweep of a cathode-ray oscillograph. The initial surge experiences a reflection on the transmission line, and the reflected wave traveling back toward the fault again experiences a reflection at the fault, which on return to the locator is recorded as a "pip" on the cathode-ray trace. The round-trip distance from locator to fault is measured by comparison with marker "pips" superimposed on the horizontal sweep.

The assistance of Mr. Howard King of the Bonneville Power Administration in obtaining test data is gratefully acknowledged.

Laurel J. Lewis is Associate Professor of Electrical Engineering at the University of Washington, Seattle, Washington, and Consulting Electrical Engineer for the Bonneville Power Administration

The type B fault locator also bases recording on the fault-generated surges, but utilizes only the initial waves arriving at the two ends of the line. One wave reaches the far end of the line, where it is picked up and transmitted back to the fault locator over a microwave communication channel, and used to start an electronic timer. The other traveling wave, reaching the fault locator directly over the transmission line actuates a delay system, which after a controlled time turns off the timer. Calibration of the controlled delay permits direct reading of the distance between fault and locator, and the result is indicated on a mechanical printer.

In the type C fault locator, the fault-generated surge is utilized only to initiate a sequence of operations, and is not recorded directly. After a controlled time delay of three cycles, a surge obtained from a charged condenser is applied to the faulted conductor and also used to initiate a sweep on a cathode-ray oscillograph. The reflected surge returning from the fault appears as a "pip" on the cathode-ray sweep.

The Linascope makes no use of the fault-generated surges, but instead is used to detect sustained faults. A periodic sequence of pulses is applied to the transmission line, and the resulting echoes are displayed on a cathode-ray tube, the sweep being synchronized at the pulse repetition rate. Tests and measurements have been made using both unidirectional pulses and modulated radio-frequency pulses.

Special Operating Problems

Actual operating results contain features not indicated by the above simple explanation. The type A locator depends on wave travel between fault and locator; there is, however, a wave traveling from the fault to the far end of the line and there being reflected. It was initially expected that the low fault-impedance would act to confine the far-end wave to that section and prevent its transmission back toward the fault locator. Experience has shown this conclusion to be incorrect; noticeable reflections have been recorded from the far end of a line with a fault at midpoint. An extra "pip" appears on the record, making interpretation difficult.

The type C locator was initially tested[2] to determine the nature of reflections under different fault conditions. It was concluded from these tests that controlled surges had to be applied to the conductor of one of the phases involved in the fault to assure reflection from the fault itself. The reasons for the types of behavior were not fully determined.

The type B locator involves only initial surges arriving at the two ends of a line, and so avoids difficulties due to multiple reflections. The system requires, however, as do the others, the use of a coupling system for obtaining pulses from the transmission line. The characteristics of the traveling waves are of importance in the design of a coupling system. Likewise the speed of wave travel is of importance in all systems, since time measurements are to be translated into distances.

In the following sections an analytical method is presented to explain some of the results not previously understood.

Mathematical Approximations

Since a complete mathematical analysis of traveling waves on a physical system is impossible, various approximate procedures are resorted to; a number of these will be reviewed to indicate the general results and to emphasize the limitations.

The simplest case is that of a two-wire line isolated in space, with losses neglected. These assumptions permit the use of inductive and capacitive coefficients and lead to the classical "wave" equation, a partial differential equation relating voltages (or currents), distance, and time. Propagation is at the velocity of light, without distortion of wave shape.

The case of a single-wire line with ground return can be treated by the same method as for the two-wire line, when the line is at a uniform height above ground and losses in the ground are neglected. Results are again of the same general form.

Efforts to include the effects of losses lead first to the introduction of series resistances and shunt conductances as line parameters. With an isolated two-wire line or a single-wire with ground return, and earth losses neglected, the mathematical formulation results in the "telegraphers' equation," the solution of which was obtained by Heaviside. A special case arises, of the so-called "distortionless" line, in which the signal velocity is the velocity of light; surges are also attenuated with distance, but do not change shape. For all other cases, however, it is impossible to speak of a single velocity, for if the initial surge is considered in terms of frequency components, each component travels at a different velocity, and with the highest frequency components approaching the velocity of light. Modification of shape is evident by lengthening of the tail of a traveling surge[4]. It is also significant that the nature of the traveling wave depends on the nature of its origin, that is, whether it is initiated by release of bound charge on the conductors or by application of a surge at a single point on the line.

That losses in line conductors are not truly represented by resistance parameters is evident when account is taken of conductor skin effect. The effect in an isolated conductor can be calculated in terms of Bessel functions[5], for a fixed frequency. The result, however, cannot be conveniently included in analysis of traveling surges, since strictly speaking the concepts of inductive and capacitive coefficients lose their validity under such conditions. The only rational approach is through the basic Maxwell equations.

One of the poorest approximations involved in the preceding methods is the assumption of perfect conduction in the earth. The situation is similar to that of skin effect in a wire. The effect for the steady state has been calculated by Carson[6]. Again the result is not one that can be included in a wave equation. Perhaps a way to include the effect would be to analyze a surge first into frequency components, and to include the effect for each component separately, and then to superimpose the results. The end result for the general case could only be expressed in terms of integral equations, which would yield little practical data.

The problem of three-phase transmission has been extensively studied; with losses neglected, waves propagate at the velocity of light. It has been shown[7] that if a surge is applied to one conductor of a uniformly transposed system, an induced surge will exist on the other two conductors, the value being determined by the potential of the point in space at which the conductor is situated. Complate reflection of the direct surge at a discontinuity results also in cancellation of the induced surges.

One of the first proposals for explaining the nature of wave distortion on three-phase lines was made by Bewley[8], who suggested a multi-velocity theory of wave travel. That waves do separate into components which travel at different velocities was convincingly demonstrated by oscillograms[9]. To explain the nature of multi-velocity components, Bewley based an analysis on a "lossless" system, with no resistance terms included specifically in the analysis, by considering that the capacitive coefficients were determined by the ground plane, and the inductive coefficients were determined by a zero plane located at a significant distance below the ground surface. That such a procedure has a basis in fact can not be denied, for the effect of finite conductivity in the earth is to result in penetration of the earth current below the surface. However, the procedure contains no basis for determining the sub-surface zero plane, nor does it account for attenuation of waves.

The fact that the use of a separate zero plane leads to terms involving two velocities of propagation tends to obscure the fundamental factors responsible for traveling wave behavior. It needs to be pointed out that the existence of multi-velocities does not depend upon such an approach, but is an inherent property of the three-phase system when losses are included. It has already been shown[10], that for the steady state, with operation at a single frequency, and even assuming a perfect earth, there are two phase velocities for a three-wire system, when conductor losses are included.

The three-phase traveling-wave system has an analogy in coupled-circuit theory. When two or more identical lumped circuits containing losses are coupled symmetrically, the system has two resonant frequencies, one higher and one lower than that of the individual mesh. A double-humped frequency response occurs for coupling larger than a critical value. The transient response contains two natural angular velocities, with one frequency higher and one lower, and with one damping factor higher and one lower, than that of the single mesh. Translated into terms of the traveling wave case, this corresponds to two velocity terms and two attenuation factors, for each frequency component traveling in each direction. From the analysis in the Appendix, the attenuation is shown to be higher for the wave traveling at the slower speed.

As for the isolated two-wire line with losses included, in a three-wire system it is again not appropriate to speak of a velocity of propagation of a surge, although it may be permissible to refer to the speed of travel of the maximum of energy distribution. It is significant that there are two such maxima traveling at different speeds. The faster wave moves at a speed higher than that on a single-phase line having the same losses, but with less attenuation and distortion.

The highest frequency components in the fast wave have velocities approaching the velocity of light. The slower wave experiences more attenuation, with the crest moving at a lower speed, although again the higher frequency components have a speed approaching that of light. The effect is a more rapid reduction in crest value and a much greater reduction in the slope of the wave front.

The Components of the Multi-Velocity Waves

The resolution of a surge into slow and fast components is accompanied by a special disposition of the currents and voltages among the three conductors. The voltages to ground of the three conductors and the three line currents may be expressed by the equations:

$$
\begin{aligned}
e_1(x,t) &= A_1 f_1(x,t,\alpha_1,v_1) + A_2 f_2(x,t,\alpha_2,v_2)\\
e_2(x,t) &= B_1 f_1(x,t,\alpha_1,v_1) + B_2 f_2(x,t,\alpha_2,v_2)\\
e_3(x,t) &= C_1 f_1(x,t,\alpha_1,v_1) + C_2 f_2(x,t,\alpha_2,v_2)\\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)\\
i_1(x,t) &= D_1 f_1(x,t,\alpha_1,v_1) + D_2 f_2(x,t,\alpha_2,v_2)\\
i_2(x,t) &= E_1 f_1(x,t,\alpha_1,v_1) + E_2 f_2(x,t,\alpha_2,v_2)\\
i_3(x,t) &= F_1 f_1(x,t,\alpha_1,v_1) + F_2 f_2(x,t,\alpha_2,v_2)
\end{aligned}
$$

These are merely symbolical expressions indicating the dependence of the voltage and current variations on separate velocity and attenuation coefficients. These coefficients themselves are not constants, but are functions of the frequency components contained in the initial wave. (A more explicit form could be used, employing Fourier integrals and the corresponding transforms.)

Of particular importance are the values of the coefficients A_1, A_2, B_1, B_2, etc. These may be functions of time, but in the simplest cases are merely numbers which determine the relative magnitudes of the individual components necessary to satisfy the initial boundary conditions. In the notation to be used:

A_1 = initial value of the fast component of e_1 (x, t)
A_2 = initial value of the slow component of e_1 (x, t)
D_1 = initial value of the fast component of i_1 (x, t)
D_2 = initial value of the slow component of i_1 (x, t)

with similar designations for the other components.

In what is to follow, a completely rigorous approach is not possible. The position taken is therefore that the losses are to be considered small, and that they can be approximated by resistance and conductance coefficients. It will also be assumed that the inductive and capacitive coefficients are to be calculated with the ground surface as the zero plane. The mathematical relations take the form of a set of equations similar to the "telegraphers' equation," but including coupling terms.

From the basic nature of the system, assuming uniformity of distributed constants and transposition of conductors, it follows, as shown in the Appendix, that:

$$
\begin{aligned}
A_1 + B_1 + C_1 &= 0\\
A_2 = B_2 &= C_2 \qquad\qquad (2)\\
D_1 + E_1 + F_1 &= 0\\
D_2 = E_2 &= F_2
\end{aligned}
$$

A similarity is to be noted between these relations and those involving symmetrical components. For example, the relation involving the slow components of current is equivalent to current flow on the three wires in parallel, with ground return, analogous to the zero sequence. The fast components of current satisfy the relations between instantaneous values in an isolated three-phase system, analogous to the sum of the positive and the negative sequences.

The relations of Equation 2 are equivalent to those obtained by Bewley,[8] but are in a sense more general, since they apply independently of the manner in which account is taken of losses. The relations hold for each frequency component and therefore to the composite.

The effect of the assumption of low loss is to make the velocities nearly independent of the losses and to make the initial ratios of slow and fast components primarily a function of the inductance and capacitance coefficients.

The slow and fast waves are each separate solutions of the differential equations. For any particular frequency component, therefore, a certain ratio exists between the voltage and current waves. This is similar to the characteristic impedance of a single-phase line. The assumption of low loss amounts to making the two characteristic impedances essentially pure resistances, and with the same essential value for each frequency component in the waves. The characteristic impedance of the fast wave is lower and that of the slow wave is higher than for a single-phase line of the same height and conductor size.

Separation of a Wave Into Components

For fault locator considerations, the important problems are the determination of the manner in which an initial surge is resolved into separate waves, the nature of their propagation, and the effect of discontinuities in the system on the transmission and reflection at these discontinuities.

The first step requires a mathematical formulation of the original surge itself. This will depend on the origin of the surge, which may result from any one of the following processes:

a) Lightning
b) Switching operations
c) Staged fault tests
d) Fault locator operation, such as in the type C equipment.

In the following discussion it will be assumed that such a formulation has been made, although it needs to be emphasized that the effects of these processes are not necessarily alike. No attempt will be made here to consider the nature of lightning itself, the effect of resulting insulator flashover, the modifying action of corona, or the non-linear nature of arcing characteristics. In the case of a staged fault where an energized system is shorted, the method of superposition may be utilized to obtain the equivalent of an applied surge, of such a form and polarity to represent the difference between the faulted condition and the original unfaulted state.

Once the initial conditions are stated, the separation of the wave into components requires the evaluation of the coefficients in Equation 1. For the twelve coefficients, six relations are provided by Equation 2. In addition, each fast wave of voltage is related to the fact wave of current by a characteristic impedance, and the slow waves are similarly related, as shown by Equation 3:

$$Z_1 = \frac{A_1}{D_1} = \frac{B_1}{E_1} = \frac{C_1}{F_1} \qquad (3)$$

$$Z_2 = \frac{A_2}{D_2} = \frac{B_2}{E_2} = \frac{C_2}{F_2}$$

Thus three initial conditions are necessary to determine the complete set of coefficients.

A large number of combinations of the six variables are possible. Solutions for the coefficients A_1, A_2, and B_1 are given in Table I for seven of the most important cases. The other nine coefficients can be derived from these by the relations of Equations 2 and 3. In Table I, the ratio of characteristic impedances is expressed by:

$$r = \frac{Z_2}{Z_1} \qquad (4)$$

It is assumed in Table I (for example in case IV) that E_A is the voltage applied to conductor No. 1, that is, e_1 (x,t) in Equation 1. Expressions for many other combinations may be obtained from Table I by cyclic rotation of the letter subscripts and the letter coefficients.

An example of the use of Table I is provided by Figure 2, which has been drawn for a step wave applied to conductor No. 1, computed with the value of r equal to 2. A positive voltage is applied to conductor No. 1, and voltages are induced on the other two conductors. The separate components satisfy the six relations of Equation 2 and also the terminal conditions.

The Propagation of the Components

From the relations derived in the Appendix, it has been determined that the surge components not only travel at different speeds, but that the slow wave is

attenuated more than the fast wave. When the attenuation is multiplied by a large distance, the effect is to make the slow wave magnitude very small in comparison to the fast component. It is concluded that in cases involving multiple reflections, the effect of the slow wave is usually negligible.

From the equations, a ratio of less than two to one might be expected for the attenuation terms in a line of usual dimensions. Actual tests on carrier systems[10] indicate that the ratio may be as high as 10 to 1 at some frequencies. It may be that slow waves flowing with a ground return experience greater losses than is indicated by the assumption of constant resistance.

Not only does the fast wave experience less attenuation than for a single-phase line, but the velocity is greater. That the fast wave is shown to travel more nearly at the velocity of light is of significance in calibration of fault locator distances in terms of time.

Another important consideration concerning calibration is related to the question of the actual path by which a wave travels, taking account of conductor sag and irregular terrain. Although no direct evidence is available from the analysis, it is tempting to speculate that since the fast-wave current flow is confined to the metallic conductors, there may be reason to believe that the distance of travel is determined primarily by the physical length of the conductor and only secondarily affected by the changing height.

The Effect of Discontinuities

Calculation of the effect of a short circuit, or other type of discontinuity can be most conveniently made by a method employing superposition. Thus, the effect of the arrival of a voltage at a short circuit can be simulated by the application of an equivalent generator, having a voltage equal and opposite to that of the arriving surge. The superposition of the original waves and those of the equivalent generator will represent the resultant conditions on both sides of the fault.

The nature of the equivalent generators to be employed to simulate a discontinuity depends on the nature of the arriving wave and the boundary conditions imposed by the discontinuity. Table I can be used to provide the relations at the equivalent generator in most cases.

An example is provided by Figure 3, illustrating a case where a surge, originally applied to phase A, as initiated in Figure 2, arrives at a fault, after sufficient travel so that the slow component has become negligible. The fault is between B and C phases and ground.

In calculating the reflected waves (denoted by primes), it is necessary to consider that the characteristic impedances are negative quantities for waves traveling in the negative direction. This is consistent with designation of positive current as a current flowing in the direction of the incident wave.

In the case of Figure 3, the continuity conditions require that the resultant conditions on the two sides of the discontinuity satisfy the following relations between transmitted waves (designated by double primes), the incident waves, and the reflected waves:

$$e_{LEFT} = e_{RIGHT} = e + e' = e''$$

$$i_{LEFT} = i + i' \qquad (5)$$

$$i_{RIGHT} = i'' = i - i'$$

These relations apply when shunt generators are used. Analogous but somewhat different relations are required when a series current-generator is used to simulate an open circuit. The relations are determined by the principle of superposition.

It is convenient to designate as transfer characteristics, the fractions of fast waves which are reflected and transmitted at a discontinuity. The voltage characteristics, for a three-phase system, unlike those for a single-phase system, are functions not only of the type of discontinuity, but also of the disposition of the incident waves. The number of possible combinations is almost without limit. However, some of the most important cases are illustrated in Table II. The values in Case 4 are those applicable to Figure 3. Although general expressions are possible in terms of the parameter r, Table II has been prepared for a value of r equal to 2, which is probably a representative value.

Special Cases

Certain special cases are of importance which require a special treatment not covered by the foregoing analysis.

The first of these is one involving dissymmetry with respect to the two directions at the fault. Two examples are illustrated in Figure 4. Neither of these are included in Table I. The analysis involves simultaneous solutions for two transmission systems having common boundary conditions, involving equal and opposite currents in the ungrounded conductors. Details of the analysis will be omitted, but the transfer characteristics are given in Table III (for r = 2), assuming an incident fast wave such as that originating in Figure 2. The fault conditions differ very little in practical form, but differ widely with respect to fault-locator considerations. The system of Figure 4(a) has a characteristic identical to that of a full three-phase fault (but only to a surge originating on phase A), while in Figure 4(b), reflected and transmitted fast waves are both several times as great as those of the incident waves themselves.

Another special case is that where the ground fault impedance is not zero. Figure 5 illustrates the form that must be assumed for the equivalent generator. The situation requires simultaneous solution of the line equations and the following equation at the generator:

$$e_1' = E_A - 2\, i_1'\, R_f \qquad (6)$$

The ratio of e_1' and i_1' can be determined by calculation from Case IV in Table I.

A third special case is illustrated in Figure 6, where the ungrounded conductors have significant lumped capacitances to ground, as would be the case where a fault occurs adjacent to a large bus structure. Figure 6(a) is applicable to the initial surge, when one conductor is suddenly grounded; Figure 6(b) is the equivalent system for simulating the effect on arrival of a surge. In these situations, the conductors act initially as though grounded, but after a time, they behave as though ungrounded. These are cases where the coefficient A_1, A_2, etc. are not constants, but are functions of time. Details will not be included, but the solutions can be considered as transitions between two states, involving a certain time constant.

Comparison with Test Results

The validity of the proposed procedure is supported by test results obtained under three widely different conditions.

Figure 7 shows a series of oscillograms previously reported by Spaulding and Diemond[2], obtained from operation of the type C fault locator, and reproduced here for illustration. Also indicated are calculations of the relative sizes of the reflections to be expected. The tests shown were all made on the Ross-Longview 230 KV line of the Bonneville Power Administration, with the line deenergized. Surges were applied at the Ross end, with faults connected at a distance of 32 miles, and with the line open at Longview, at a distance of 51 miles. These are all cases represented by Cases 1 through 8 in Table II. Some of the results show reflections obtained from the fault and from the far end, while others show reflections from the fault only and others from the far end only. Reflections indicated at 62 miles are reflections obtained from the fault after two round trips. Considering that any reflections obtained from the far end must traverse the fault position twice, once in each direction, it is necessary to multiply the incident wave twice by the transmission coefficient in order to obtain comparative results. Since the fault-locator response is dependent on the steepness of the wave front, which changes due to attenuation, a direct comparison between reflections obtained at the fault and at the far end is not applicable. If it be assumed that the steepness of the wave is diminished approximately in proportion to distance traveled, and proper allowance made, the results obtained are almost exactly those which are indicated by Table II, within the limits of recording accuracy.

Another series of tests, made on the Bonneville system in February 1951, included two from which the records in Figures 8 and 9 were obtained. In each case, a fault was initiated by closing a circuit breaker at Chehalis substation, 87 miles from Ross substation, where the records were obtained. For Figure 8, the fault was from A to B phase, and for Figure 9, from B phase to ground. The records of Figure 8(a) and 9(a) are indicative of the true nature of the traveling waves, and were obtained by recording the voltage on B phase at Ross through a coupling system consisting of a high-voltage carrier coupling capacitor and a voltage divider network. The coupling system introduced a long time constant of about 450 microseconds, about half the length of the record, which modified the long-time average but did

not materially modify the short-time variations. The records in Figure 8(b) and 9(b) were taken on the type A fault locator equipment (but with some modifications not of significance here.) The input to the fault locator was obtained from an antenna-type coupler, with a very short time constant, located under the main station bus. The effect of the short time constant is to make the response essentially proportional to the rate of change of the input recorded by the first oscillograph.

Although the records of Figures 8 and 9 show reflections obtained from other parts of the system, particular attention is directed to the differences in shape of the initial surges at the beginning of the records, and the magnitudes of the reflections at the 87-mile point (the range markers are at 10-mile intervals). A line-to-ground fault initiates a wave consisting of both slow and fast components; in the present instance, the initial surge was modified by the effect of a large capacitance to ground on the unfaulted conductors, such as was discussed in connection with Figure 6, The initial wave rises to a maximum and then actually decreases, with a time constant determined by the bus capacitance and the line impedances (the effect is somewhat obscured, however, because of the capacitance of the bus structure at the receiving end). In the line-to-line fault, only a fast-wave component is produced; since no voltage change occurs on the unfaulted conductor, its capacitance to ground has no effect. The difference between reflected waves at the 87-mile point is the result of two factors: first, the line-to-line fault produces an initial fast wave of larger value, and second, the reflection characteristics are different in the two cases. For the line-to-line fault, the reflection coefficient is 100 per cent; for the line-to-ground fault, the reflection coefficient would be 100 per cent at the initial instant because of the capacitance, but if the bus capacitance were not considered, the coefficient would be only 50 per cent. The actual results correspond to a value somewhere between. It is to be expected that after the wave front of a surge has become less steep due to attenuation, the modifying action of the bus capacitance will become less pronounced.

A third set of data for comparison is provided by an interesting series of oscillograms published by Rudge[11] in 1930., These were obtained for different types of discontinuities and recording conditions by application of surges to phase A and measurements of voltage on phase B at points approximately 5 miles away. These are examples in which the slow waves are of significant magnitude, but the differences in velocity have resulted in the separation of components. Although the information available does not permit quantitative analysis, the application of the principles discussed here give predictions of wave shape in full agreement with test results. Account must be taken not only of the velocity differences, but also of the reduction in slope of the slow wave.

Conclusions

An appraisal of the response possibilities for the different fault locators can be made in terms of the foregoing analysis. From the standpoint of the initial waves, the type C and the Linascope are the simplest systems, since the surges are controlled and of the same character each time. Because of the relative unimportance of the slow wave, an applied-surge system is effective only to the

extent of the fast wave produced, and only if the fast wave is such as to produce reflections for all types of faults to be recorded. Applying a single voltage always to the same conductor does not insure reflections. The selective switching arrangement of the type C system does assure reflections; an alternate arrangement would be the application of unequal voltages simultaneously between each of two conductors and ground. In the latter arrangement, by the proper choice of the voltages it would be possible to establish fast wave voltages between each conductor and ground and between each pair of conductors, so that reflections would be obtained from both line-to-ground and line-to-line faults.

In the type A and B systems, the initial surge will have a character determined by the nature of the fault. Particularly in the case of lightning strokes, it would appear most probable that a fault would be initially from one line to ground. For the type A system, a subsequent reflection from the fault might involve surges of a different nature than the original, particularly if the initial fault condition has progressed to a multi-line fault. At least the original faulted phase would still be involved in the discontinuity. There is reason to believe that some of the conditions possible in staged tests would be unlikely to occur with lightning strokes.

A major concern in all systems is the type of coupling used to record traveling waves. The coupling system should have a response with as little variability as possible. With recording dependent on fast waves, the possible combinations of received voltage are the same as the possible combinations of instantaneous voltage in a three-phase system. Thus, coupling to a single conductor alone does not assure a response. The most effective system would be one which would respond to the largest of the three voltages which could exist on any one of the three conductors at any one instant.

MATHEMATICAL APPENDIX

The voltage and current variations for a three-wire transmission line may be considered as represented by:

$$
\begin{aligned}
-\frac{\partial e_1}{\partial x} &= L_{11}\frac{\partial i_1}{\partial t} + R_{11}i_1 + L_{12}\frac{\partial i_2}{\partial t} + L_{13}\frac{\partial i_3}{\partial t} \\
-\frac{\partial e_2}{\partial x} &= L_{21}\frac{\partial i_1}{\partial t} + L_{22}\frac{\partial i_2}{\partial t} + R_{22}i_2 + L_{23}\frac{\partial i_3}{\partial t} \qquad (1) \\
-\frac{\partial e_3}{\partial x} &= L_{31}\frac{\partial i_1}{\partial t} + L_{32}\frac{\partial i_2}{\partial t} + L_{33}\frac{\partial i_3}{\partial t} + R_{33}i_3
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\partial i_1}{\partial x} &= C_{11}\frac{\partial e_1}{\partial t} + G_{11}e_1 + C_{12}\frac{\partial e_2}{\partial t} + C_{13}\frac{\partial e_3}{\partial t} \\
-\frac{\partial i_2}{\partial x} &= C_{21}\frac{\partial e_1}{\partial t} + C_{22}\frac{\partial e_2}{\partial t} + G_{22}e_2 + C_{23}\frac{\partial e_3}{\partial t} \qquad (2) \\
-\frac{\partial i_3}{\partial x} &= C_{31}\frac{\partial e_1}{\partial t} + C_{32}\frac{\partial e_2}{\partial t} + C_{33}\frac{\partial e_3}{\partial t} + G_{33}e_3
\end{aligned}
$$

Symmetry conditions will be assumed, as a result of uniformity of coefficients and transpositions. Inductive and capacitive coefficients may be evaluated in accordance with methods previously described [12]. It will be assumed that the ground surface is the zero plane.

Using $E_1(x,s)$, or simply E_1, as the Laplace transform of $e_1(x,t)$, and similar notation for the other terms, and neglecting initial conditions for simplicity, the set of equations may be written as:

$$-\frac{\partial E_n}{\partial x} = \sum_{r=1}^{3} J_{nr}(s)\, I_r \qquad (3)$$

$$-\frac{\partial I_n}{\partial x} = \sum_{r=1}^{3} K_{nr}(s)\, E_r \qquad (4)$$

where: $J_{nn}(s) = J = sL_{nn} + R_{nn} = sL + R$
$J_{nm}(s) = J' = sL_{nm} = sL'$
$K_{nn}(s) = K = sC_{nn} + G_{nn} = sC + G$
$K_{nm}(s) = K' = sC_{nm} = sC'$

Differentiating Equation 3 and substituting from Equation 4, there results:

$$\frac{\partial^2 E_n}{\partial x^2} = \sum_{r=1}^{3} M_{nr}(s)\, E_r$$

and similarly:

$$\frac{\partial^2 I_n}{\partial x^2} = \sum_{r=1}^{3} M_{nr}(s)\, I_r \qquad (6)$$

where $M_{nr}(s) = \sum_{r=1}^{3}\sum_{q=1}^{3} J_{nr}(s)\, K_{rq}(s)$

which will be designated briefly by:

$$M_{nn}(s) = M$$
$$M_{nm}(s) = M'$$

Denoting $\frac{\partial^2 E_n}{\partial x^2}$ by $p^2 E_n$, the solution for the complex propagation coefficients $p_n = \alpha_n(s) + \beta_n(s)$ depends on the roots of the characteristic equation:

$$\begin{vmatrix} [p^2 - M] & -M' & -M' \\ -M' & [p^2 - M] & -M' \\ -M' & -M' & [p^2 - M] \end{vmatrix} = 0$$

which can also be expressed as:

$$[p^2 - (M - M')]^2\, [p^2 - (M + 2M')] = 0$$

or alternately as

$$[p^2 - (K - K')(J - J')]^2\, [p^2 - (K + 2K')(J + 2J')] = 0$$

Of the four propagation coefficients, two apply to the wave moving in the positive direction. Solutions of Equation 5 for the positive wave are:

$$E_n(s,x) = F_{n1}(s)\, e^{\alpha_1(s)\cdot x}\, e^{\beta_1(s)\cdot x} + F_{n2}(s)\, e^{\alpha_2(s)\cdot x}\, e^{\beta_2(s)\cdot x} \qquad (8)$$

The expressions for $F_{n1}(s)$ and $F_{n2}(s)$ are transforms of time functions which are to be evaluated for conditions existing at $x = 0$, and in many cases will have the same form as $E_n(s,0)$ except for arbitrary constants.

Solutions to the original Equations 1 are then obtained by taking the inverse Laplace transforms of Equation 8. The results are expressed symbolically as:

$$
\begin{aligned}
e_1(x,t) &= A_1 f_1(x,t,\alpha_1,v_1) + A_2 f_2(x,t,\alpha_2,v_2) \\
e_2(x,t) &= B_1 f_1(x,t,\alpha_1,v_1) + B_2 f_2(x,t,\alpha_2,v_2) \\
e_3(x,t) &= C_1 f_1(x,t,\alpha_1,v_1) + C_2 f_2(x,t,\alpha_2,v_2)
\end{aligned} \tag{9}
$$

Similar expressions for the currents are:

$$
\begin{aligned}
i_1(x,t) &= D_1 f_1(x,t,\alpha_1,v_1) + D_2 f_2(x,t,\alpha_2,v_2) \\
i_2(x,t) &= E_1 f_1(x,t,\alpha_1,v_1) + E_2 f_2(x,t,\alpha_2,v_2) \\
i_3(x,t) &= F_1 f_1(x,t,\alpha_1,v_1) + F_2 f_2(x,t,\alpha_2,v_2)
\end{aligned} \tag{10}
$$

An alternate view to Equations 3 through 8 is obtained through comparison of the Laplace transform with the related Fourier transform. Thus, by writing ($j\omega$) in place of (s) in Equation 8 there is obtained a relationship applicable to each value of frequency in the frequency spectrum of the traveling wave. The phase characteristic at each frequency is related to the phase velocity by:

$$
v(\omega) = \frac{\omega}{\beta(\omega)} \tag{11}
$$

Equations 9 and 10 can likewise be considered as summations through Fourier integrals of the frequency components of Equation 7.

Substituting Equations 9 in Equations 5, and Equations 10 in Equations 6, and treating each velocity term separately, using the relations of Equation 7, there results:

$$
\begin{aligned}
A_1 + B_1 + C_1 &= 0 \\
A_2 = B_2 &= C_2 \\
D_1 + E_1 + F_1 &= 0 \\
D_2 = E_2 &= F_2
\end{aligned} \tag{12}
$$

In terms of the frequency variable, there is obtained by Equations 5 and 12:

$$
\begin{aligned}
A_1\left[\alpha_1(\omega) + j\beta_1(\omega)\right] &= D_1 \left[J(j\omega) - J'(j\omega)\right] \\
A_2\left[\alpha_2(\omega) + j\beta_2(\omega)\right] &= D_2 \left[J(j\omega) + 2\,J'(j\omega)\right]
\end{aligned} \tag{13}
$$

To the extent that the losses are small, approximate expressions may be obtained for the ratios in Equation 13:

$$
\begin{aligned}
Z_1 = \frac{A_1}{D_1} &\approx v_1(L - L') \\
Z_2 = \frac{A_2}{D_2} &\approx v_2(L + 2L')
\end{aligned} \tag{14}
$$

Values of the complex propagation coefficients are obtained from Equation 7, and with the higher order terms neglected, may be expressed in the form:

$$
p(s) = \sqrt{\beta_0^2 s^2 \left[1 + \frac{a}{s}\right]} \tag{15}
$$

where $\beta_0 = \frac{1}{c}$

and c = velocity of light

or in terms of frequency components, as

$$
\begin{aligned}
p(j\omega) &= \alpha(\omega) + j\beta(\omega) \\
&\approx \frac{a_n}{2}\beta_0 + j\omega\beta_0 \sqrt{1 + \left(\frac{a_n}{\omega}\right)^2}
\end{aligned}
$$

For the individual terms, the attenuation parameters become:

$$
\begin{aligned}
a_1 &= \frac{R}{L - L'} + \frac{G}{C - C'} \\
a_2 &= \frac{R}{L + 2L'} + \frac{G}{C + 2C'}
\end{aligned} \tag{16}
$$

The individual velocity terms, from Equation 11, are:

$$
\begin{aligned}
v_1 &\approx \frac{c}{\sqrt{1 + \left(\frac{a_1}{\omega}\right)^2}} \\
v_2 &\approx \frac{c}{\sqrt{1 + \left(\frac{a_2}{\omega}\right)^2}}
\end{aligned} \tag{17}
$$

1. "A Transmission Line Fault Locator Using Fault Generated Surges," R. F. Stevens, T. W. Stringfield, AIEE Trans. Vol. 67, 1948, pp. 1168-78.

2. "A Transient Fault Locator for High Voltage Transmission Lines," L. R. Spaulding, C. C. Diemond, AIEE Trans. Vol. 68, 1949, pp. 1005-10.

3. The "Linascope" - An Echo Ranging Type Fault Locator for High Voltage Lines, J. R. Leslie, K. H. Kidd, AIEE Trans. Vol. 67, 1949, pp. 1162-67.

4. Electromagnetic Theory (book), J. A. Stratton. McGraw-Hill Book Co., New York, 1941, pp. 297-309.

5. Principles of Electric Power Transmission (book), L. R. Woodruff. John Wiley and Sons, Inc., New York, 1925, pp. 78-86.

6. "Wave Propagation in Overhead Wires with Ground Return," J. R. Carson, Bell System Technical Journal, Oct. 1926, p. 539.

7. "Traveling Waves on Transmission Systems," L. V. Bewley. AIEE Trans., Vol. 50, 1931, pp. 532-50.

8. "Attenuation and Distortion of Waves," L. V. Bewley. AIEE Trans., Vol. 53, 1934, pp. 1749-57.

9. "Resolution of Surges Into Multi-Velocity Components," L. V. Bewley. AIEE Trans., Vol. 54, 1935, pp. 1199-1203.

10. "Propagation of High-frequency Waves Along a Symmetrical Three-phase Line, A. Chevalier. Revue Generale De L'Electricte, Paris, Jan. 1945.

11. "Traveling Waves on Transmission Lines with Artificial Lightning Surges," Mc Eachron, Hemstreet, Rudge. Discussion by W. J. Rudge. AIEE Trans. Vol., 49, 1930, pp. 932-33.

12. "Critique of Ground Wire Theory," L. V. Bewley. AIEE Trans. Vol., 50, 1931 pp. 1-22.

TABLE I

SURGE COMPONENTS FOR SPECIFIED CONDITIONS

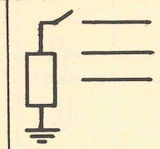
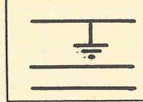
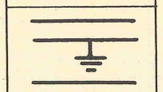
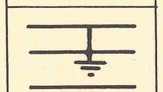
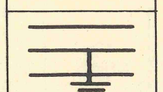
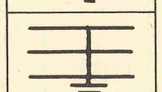
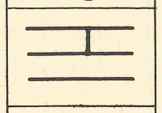
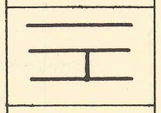
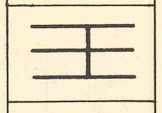
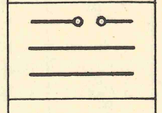
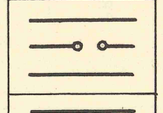
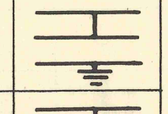
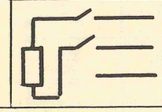
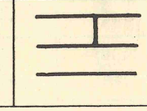
CASE	INITIAL CONDITIONS			A_1	B_1	A_2
I	E_A	E_B	E_C	$\frac{1}{3}(2E_A - E_B - E_C)$	$\frac{1}{3}(-E_A + 2E_B - E_C)$	$\frac{1}{3}(E_A + E_B + E_C)$
II	I_A	I_B	I_C	$\frac{Z_1}{3}(2I_A - I_B - I_C)$	$\frac{Z_1}{3}(-I_A + 2I_B - I_C)$	$\frac{Z_1}{3}(I_A + I_B + I_C)$
III	E_A	E_B	I_C	$\frac{1}{1+2r}\left[(1+r)E_A - rE_B - Z_2I_C\right]$	$\frac{1}{1+2r}\left[-rE_A + (1+r)E_B - Z_2I_C\right]$	$\frac{r}{1+2r}\left[E_A + E_B + \frac{1}{r}Z_1I_C\right]$
IV	E_A	I_B	I_C	$\frac{1}{2+r}\left[2E_A - Z_2I_B - Z_2I_C\right]$	$\frac{1}{2+r}\left[-E_A + (1+r)Z_1I_B - Z_1I_C\right]$	$\frac{1}{2+r}\left[rE_A + Z_2I_B + Z_2I_C\right]$
V	E_{AB}	E_{BC}	E_{CA}	$\frac{1}{3}(2E_{AB} + E_{BC})$	$\frac{1}{3}(2E_{BC} + E_{CA})$	0
VI	E_{AB}	I_{AB}	I_C	$\frac{1}{2}E_{AB} + \frac{1}{6}Z_1I_{AB} - \frac{1}{3}Z_1I_C$	$-\frac{1}{2}E_{AB} + \frac{1}{6}Z_1I_{AB} - \frac{1}{3}Z_1I_C$	$\frac{1}{3}Z_2(I_{AB} + I_C)$
VII	E_{AB}	I_{AB}	E_C	$\frac{1}{2}E_{AB} - \frac{1}{2+r}E_C + \frac{1}{4+2r}Z_2I_{AB}$	$-\frac{1}{2}E_{AB} - \frac{1}{2+r}E_C + \frac{1}{4+2r}Z_2I_{AB}$	$\frac{1}{2+r}\left[rE_C + Z_2I_{AB}\right]$

$r = \frac{Z_2}{Z_1}$

$E_{AB} = E_A - E_B$ $\qquad I_{AB} = I_A + I_B$

TABLE II

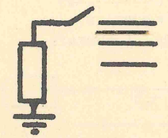
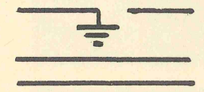
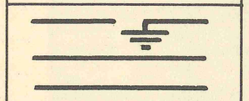
FAST-WAVE VOLTAGE TRANSFER COEFFICIENTS *

CASE	APPLIED SURGE	SYSTEM	GENERATORS		VOLTAGE TRANSFER COEFFICIENTS		
			CASE	DATA	PHASE	REFLECTION	TRANSMISSION
1			IV	E_A $I_B = 0$ $I_C = 0$	A B C	- 0.5 - 0.5 - 0.5	0.5 0.5 0.5
2			IV	E_B $I_A = 0$ $I_C = 0$	A B C	- 0.125 - 0.5 0.25	0.875 0.5 1.25
3			III	E_A E_B $I_C = 0$	A B C	- 0.8 - 1.4 - 0.2	0.2 - 0.4 0.8
4			III	E_B E_C $I_A = 0$	A B C	- 0.2 - 0.2 - 0.2	0.8 0.8 0.8
5			I	E_A E_B E_C	A B C	- 1.0 - 1.0 - 1.0	0 0 0
6			VI	E_{AB} $I_{AB} = 0$ $I_C = 0$	A B C	- 0.75 - 1.5 0	0.25 - 0.5 1.0
7			VI	E_{BC} $I_{BC} = 0$ $I_A = 0$	A B C	0 0 0	1.0 1.0 1.0
8			V	E_{AB} E_{BC} E_{CA}	A B C	- 1.0 - 1.0 - 1.0	0 0 0
9			II	I_A $I_B = 0$ $I_C = 0$	A B C	0.8 0.8 0.8	0.2 0.2 0.2
10			II	I_B $I_A = 0$ $I_C = 0$	A B C	0.2 0.8 - 0.4	0.8 0.2 1.4
11			VII	E_{AB} $I_{AB} = 0$ E_C	A B C	- 0.875 - 1.25 - 0.5	0.125 0.25 0.5
12			VI	E_{AB} $I_{AB} = 0$ $I_C = 0$	A B C	- 1.0 - 1.0 - 1.0	0 0 0

* Computed for $r = \frac{Z_2}{Z_1} = 2$

TABLE III

FAST-WAVE VOLTAGE TRANSFER COEFFICIENTS *

APPLIED SURGE	SYSTEM	VOLTAGE TRANSFER COEFFICIENTS		
		PHASE	REFLECTION	TRANSMISSION
		A B C	- 1.0 - 1.0 - 1.0	0 0 0
		A B C	3.0 3.0 3.0	4.0 4.0 4.0

* Computed for $r = \frac{Z_2}{Z_1} = 2$

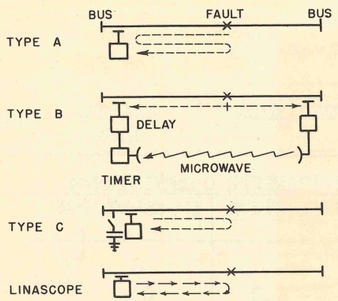


FIGURE 1
FAULT LOCATOR OPERATION

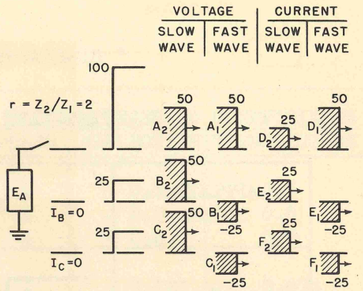


FIGURE 2
SURGE COMPONENTS

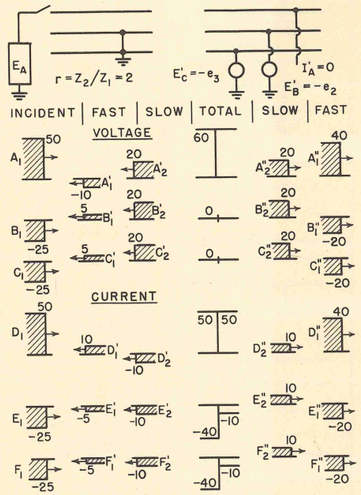


FIGURE 3 — WAVE COMPONENTS

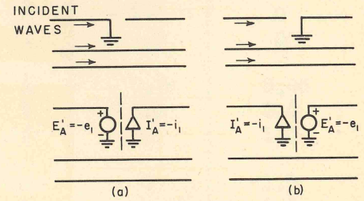


FIGURE 4
FAULT REPRESENTATION

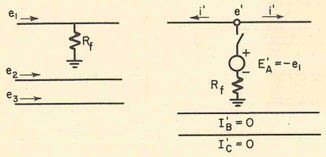


FIGURE 5
FAULT WITH GROUND RESISTANCE

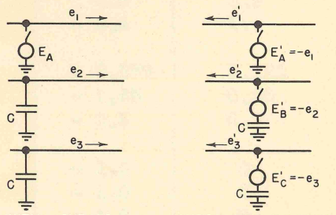


FIGURE 6
FAULT WITH CAPACITANCE TO GROUND

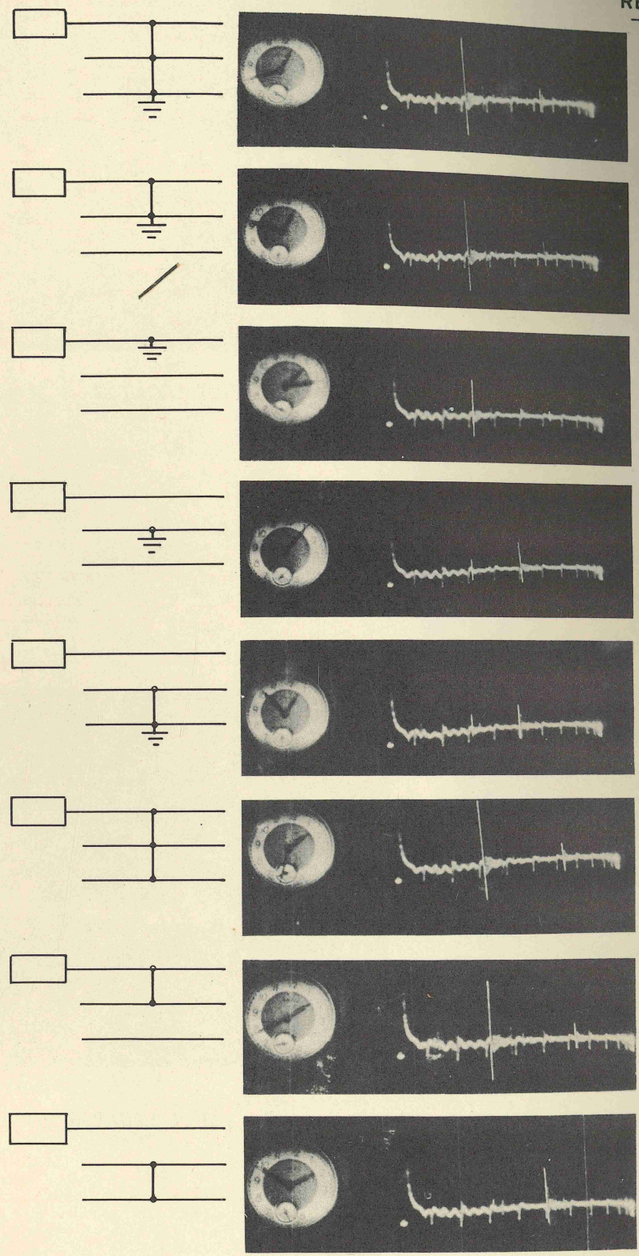
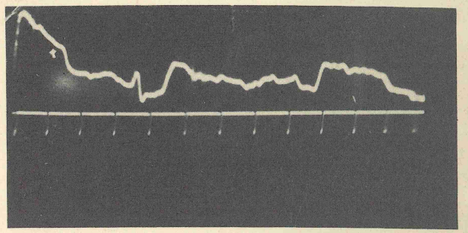
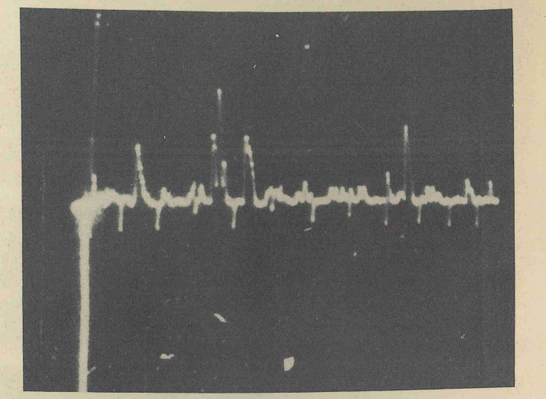


FIGURE 7
TYPE C DEAD LINE TESTS



VOLTAGE ON B PHASE

(a)

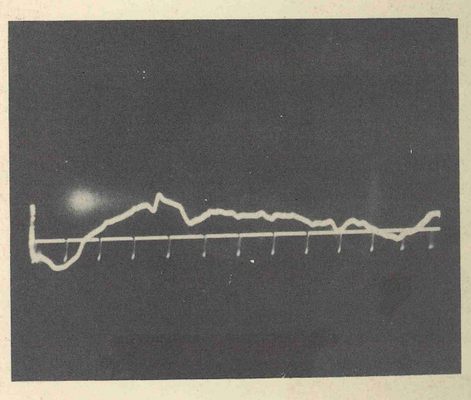


FAULT LOCATOR

(b)

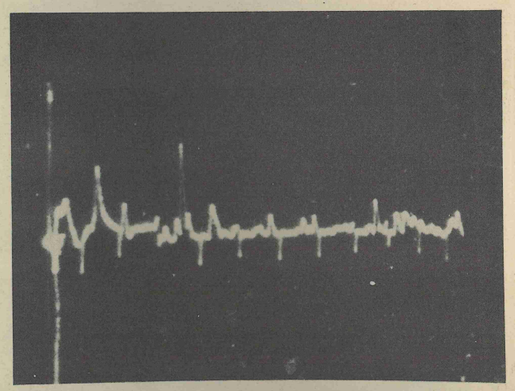
FIGURE 8

FAULT FROM A TO B PHASE
RECORDS TAKEN AT 87 MILES



VOLTAGE ON B PHASE

(a)



FAULT LOCATOR

(b)

FIGURE 9

FAULT FROM B PHASE TO GROUND

AIEE
Technical Paper
51-312

July 1951
Litho in USA

Equip. - Testing

TRAVELING WAVE RELATIONS APPLICABLE TO POWER-SYSTEM FAULT LOCATORS

Laurel J. Lewis
Member AIEE
University of Washington
Seattle, Wash.

Advance Copy
Not Released for Publication

All Rights Reserved by the
American Institute of Electrical Engineers
33 West 39th Street, New York 18, N.Y.

A paper recommended by the AIEE Committee on Transmission and Distribution and approved by the AIEE Technical Program Committee for presentation at the AIEE Pacific General Meeting, Portland, Oregon, August 20-23, 1951. Manuscript submitted May 22, 1951; made available for printing July 10,1951.

PRICE: 30¢ To Members
60¢ To Nonmembers